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Approximate Expression for Interaction of a Two Similar Plane Double Layer at Moderate and Low Potential

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The interaction energy between two similar plane parallel double layers is expanded in a series of $\tanh y_0/4$ and a general expression is introduced. The series converges very fast at moderate and low potentials. With the first four terms of the series a very high precision with six significant figures can be reached when the dimensionless surface potential y_0 of the colloidal particles is less or equal to 3.

KEY WORDS: plane parallel double layers, interaction energy, colloid particles

1. Introduction

To calculate the interaction energy between two similar plane parallel double layers is to find elliptic integrals finally. In order to obtain approximate expressions for the interaction, one usually expands the interaction energy in the power series of the dimensionless surface potential y_0. However this expansions converge very slowly. Honig and Mul [1] suggested that better approximations could be obtained if the interaction energy was expressed as a series function of $\tanh(y_0/4)$ instead of y_0. Ohshima and Healy et al., as well as Ohshima and Kondo [2–3] had derived some approximate expressions by such method. But their method is comparing – coefficient method. For this method not only is workload larger, but also only first several terms of a series were obtained. In following discussion, we expand the elliptic integrals related to the interaction energy in a series of $\tanh(y_0/4)$, and hereby find the interaction energy of two similar plane parallel double layers. So far my method has not been used by anyone yet.

2. Interaction between two plates

Supposing that a layer of a symmetrical electrolyte solution infinite in two dimension but of variable thickness d is confined between two parallel and planar surfaces (see figure 1), the potentials on two plates are constant $\varphi_0(>0)$, and the potential between two plates is φ, and φ has one minimum point $\left(\frac{d}{2}, \varphi_e\right)$, the distance from left plate is x

For convenience we introduce the following dimensionless parameters

$$y = \frac{ze\varphi}{kT}, \qquad \xi = \kappa x, \tag{1}$$

$$p' = \frac{p}{2nkT} = \frac{z^2e^2p}{\varepsilon\kappa^2\,(kT)^2}, \tag{2}$$

$$V' = \frac{\kappa V}{2nkT} = \frac{z^2e^2V}{\varepsilon\kappa\,(kT)^2}, \tag{3}$$

where z is the charge number of the positive ion, e is the proton charge, k is Boltzmann constant, T is the absolute temperature, $\kappa=\sqrt{(8\pi ne^2z^2)/(\varepsilon kT)}$ is Debye parameter, p is the repulsive force between two plates, n is the ion concentration of the bulk solution, ε is the dielectric constant of the solution, V is the repulsive energy per unit area between two plates. p' and V' is [4]

$$p' = \cosh y_e - 1, \tag{4}$$

$$V' = \int_{\xi_d}^{\infty} p'\,\mathrm{d}\xi_d = \int_{\xi_d}^{\infty} (\cosh y_e - 1)\,\mathrm{d}\xi_d, \tag{5}$$

where $2\cosh y_e = -C$, and C is integral constant. ξ_d is [4]

$$\begin{aligned}
\xi_d &= \sqrt{2}\int_{y_e}^{y_0} \frac{\mathrm{d}y}{\sqrt{\cosh y - \cosh y_e}} \\
&= \sqrt{2}\int_{y_e}^{y_0} \frac{\sinh y\mathrm{d}y}{\sinh y\sqrt{\cosh y - cosh y_e}} \\
&= \int_{\cosh y_e}^{\cosh y_0} \frac{\sqrt{2}\mathrm{d}\cosh y}{\sqrt{\cosh y - \cosh y_e}\sqrt{\cosh^2 y - 1}}.
\end{aligned} \tag{6}$$

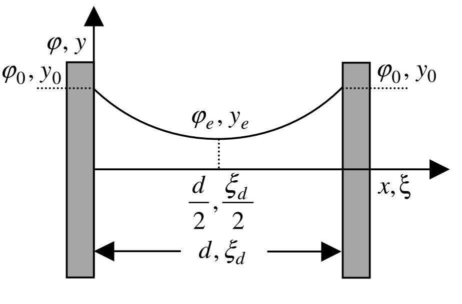

Figure 1. Interaction between two plates.

Equation (5) can be rewritten as follows:

$$
\begin{aligned}
V' = &\ (\cosh y_e - 1)\xi_d|_{\xi_d}^{\infty} - \int_{y_e}^{0} \xi_d \sinh y_e \mathrm{d}y_e \\
= &\lim_{\xi_d \to \infty} (\cosh y_e - 1)\xi_d \\
&-(\cosh y_e - 1)\xi_d + \int_{0}^{y_e} \xi_d \sinh y_e \mathrm{d}y_e,
\end{aligned} \tag{7}
$$

when $\xi_d \to \infty$, $y_e \to 0$, so the first term on the right of equation (7) is indefinite. By equation (6), the first term on the right of equation (7) can be rewritten as

$$
\begin{aligned}
\lim_{\xi_d \to \infty} (\cosh y_e - 1)\xi_d &= -\lim_{C \to -2} \int_{-\frac{C}{2}}^{\cosh y_0} \frac{(2 + C)\mathrm{d} \cosh y}{\sqrt{\cosh^2 y - 1}\sqrt{2\cosh y + C}} \\
&= -\lim_{C \to -2} \int_{-\frac{C}{2}}^{\cosh y_0} \frac{\sqrt{2\cosh y + C}\, \mathrm{d} \cosh y}{\sqrt{\cosh y - 1}\sqrt{\cosh y + 1}} \\
&\quad + \lim_{C \to -2} \int_{-\frac{C}{2}}^{\cosh y_0} \frac{2\sqrt{\cosh y - 1}\, \mathrm{d} \cosh y}{\sqrt{\cosh y + 1}\sqrt{2\cosh y + C}} \\
&= -\int_{1}^{\cosh y_0} \frac{\sqrt{2}\, \mathrm{d} \cosh y}{\sqrt{\cosh y + 1}} + \int_{1}^{\cosh y_0} \frac{\sqrt{2}\, \mathrm{d} \cosh y}{\sqrt{\cosh y + 1}} \to 0,
\end{aligned}
$$

so equation (7) becomes

$$
\begin{aligned}
V' &= -(\cosh y_e - 1)\xi_d + \int_{0}^{y_e} \xi_d \sinh y_e \, \mathrm{d}y_e \\
&= -(\cosh y_e - 1)\xi_d + \int_{0}^{y_e} \left(\int_{y_e}^{y_0} \frac{\sqrt{2}\, \mathrm{d}y}{\sqrt{\cosh y - \cos y_e}} \right) \sinh y_e \mathrm{d}y_e.
\end{aligned} \tag{8}
$$

Changing the order of the integral in equation (8) (see figure 2), we obtain

$$
\begin{aligned}
V' &= -(\cosh y_e - 1)\xi_d + \sqrt{2} \int_{0}^{y_e} \left(\int_{0}^{y} \frac{\sinh y_e \, \mathrm{d}y_e}{\sqrt{\cosh y - \cosh y_e}} \right) \mathrm{d}y \\
&\quad + \sqrt{2} \int_{y_e}^{y_0} \left(\int_{0}^{y_e} \frac{\sin y_e \, \mathrm{d}y_e}{\sqrt{\cosh y - \cosh y_e}} \right) \mathrm{d}y \\
&= -(\cosh y_e - 1)\xi_d - \sqrt{2} \int_{0}^{y_e} \left[\int_{0}^{y_e} \frac{\mathrm{d}(\cosh y - \cosh y_e)}{\sqrt{\cosh y - \cosh y_e}} \right] \mathrm{d}y \\
&\quad - \sqrt{2} \int_{y_e}^{y_0} \left[\int_{0}^{y_e} \frac{\mathrm{d}(\cosh y - \cosh y_e)}{\sqrt{\cosh y - \cosh y_e}} \right] \mathrm{d}y,
\end{aligned}
$$

viz.

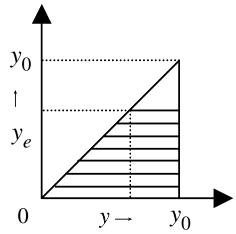

Figure 2. The integral region of the interaction energy.

$$
\begin{aligned}
V' &= -(\cosh y_e - 1)\xi_d + 2\sqrt{2}\int_0^{y_e}\sqrt{\cosh y - 1}\,\mathrm{d}y \\
&\quad +2\sqrt{2}\int_{y_e}^{y_0}\left(\sqrt{\cosh y - 1} - \sqrt{\cosh y - \cosh y_e}\right)\mathrm{d}y \\
&= -(\cosh y_e - 1)\,\xi_d + 2\sqrt{2}\int_0^{y_0}\sqrt{\cosh y - 1}\mathrm{d}y \\
&\quad -2\sqrt{2}\int_{y_e}^{y_0}\sqrt{\cosh y - \cosh y_e}\mathrm{d}y \\
&= -(\cosh y_e - 1)\,\xi_d + 2\sqrt{2}\int_1^{\cosh y_0}\frac{\sqrt{\cosh y - 1}}{\sqrt{\cosh^2 y - 1}}\mathrm{d}\cosh y \\
&\quad -2\sqrt{2}\int_{y_e}^{y_0}\sqrt{\cosh y - \cosh y_e}\mathrm{d}y \\
&= -(\cosh y_e - 1)\,\xi_d + \int_1^{\cosh y_0}\frac{2\sqrt{2}\mathrm{d}\cosh y}{\sqrt{\cosh y + 1}} \\
&\quad -2\sqrt{2}\int_{y_e}^{y_0}\sqrt{\cosh y - \cosh y_e}\mathrm{d}y \\
&= -(\cosh y_e - 1)\,\xi_d + 4\sqrt{2}\left(\sqrt{\cosh y_0 + 1} - \sqrt{2}\right) \\
&\quad -2\sqrt{2}\int_{y_e}^{y_0}\sqrt{\cosh y - \cosh y_e}\mathrm{d}y. \qquad (9)
\end{aligned}
$$

Substituting equations (6) into (9), we obtain

$$
\begin{aligned}
V' = {}& 4\sqrt{2}\left(\sqrt{\cosh y_0 + 1} - \sqrt{2}\right) - \int_{\cosh y_e}^{\cosh y_0}\frac{\sqrt{2}\,(\cosh y_e - 1)\,\mathrm{d}\cosh y}{\sqrt{\cosh^2 y - 1}\sqrt{\cosh y - \cosh y_e}} \\
&-2\sqrt{2}\int_{\cosh y_e}^{\cosh y_0}\frac{\sqrt{\cosh y - \cosh y_e}}{\sqrt{\cosh^2 y - 1}}\mathrm{d}\cosh y. \qquad (10)
\end{aligned}
$$

The integrals on the right of equations (6) and (10) are elliptic integrals, now we find the integrals by expanding them in series. We find ξ_d first. Let $\tanh(y/4) = u$, we have

$$\cosh y = \frac{8u^2}{\left(1 - u^2\right)^2} + 1, \tag{11}$$

$$\cosh y_e = \frac{8u_e^2}{\left(1 - u_e^2\right)^2} + 1, \tag{12}$$

or

$$u_e = \frac{\sqrt{\cosh y_e - 1}}{\sqrt{\cosh y_e + 1} + \sqrt{2}} = \frac{\sqrt{-C - 2}}{\sqrt{-C + 2} + 2}. \tag{13}$$

Substituting equations (11) and (12) into equation (6), we obtain

$$\xi_d = 2\left(1 - u_e^2\right) \int_{u_e}^{u_0} \frac{\mathrm{d}u}{\sqrt{u^2 - u_e^2}\sqrt{1 - u_e^2 u^2}}, \tag{14}$$

where $u_0 = \tanh(y_0/4)$. Expanding ξ_d in the series, and retaining the first n terms, we obtain approximate expression

$$\xi_{d,n} = 2\left(1 - u_e^2\right) \sum_{m=0}^{n} \frac{(2m)! u_e^{2m}}{4^m (m!)^2} I_m, \tag{15}$$

where

$$I_m = \int_{u_e}^{u_0} \frac{u^{2m}\, \mathrm{d}u}{\sqrt{u^2 - u_e^2}} \quad (m = 0, 1, 2, \ldots). \tag{16}$$

Equation (15) is just the expression used to calculate the distance between two plates.

By equation (14), we know that there are following some strongpoint for expanding ξ_d in the series of $u_e u$: first, y in $\tanh(y/4)$ is divided by 4, this is equivalent to reduce y; second, because every term in the series consists of the products of u_e and u, when both y and y_e are less viz. both u_e and u are less, the second term of the series is the fourth little quantity, the third term is the eighth little quantity, and so on, so when both y and y_e are less, the series converges very fast. Again, whether how large both y and y_e are, the absolute value of both u_e and u is always less than 1, and they monotonously increase with y and y_e, thus the series is always convergent.

Now we find I_m. Let $\frac{u_e}{u} = \cos\theta$, equation (16) becomes

$$I_m = \int_{u_e}^{u_0} \frac{u^{2m-1}\mathrm{d}u}{\sqrt{1 - \frac{u_e^2}{u^2}}} = u_e^{2m} \int_0^{\theta_0} \frac{\mathrm{d}\theta}{\cos^{2m+1}\theta}$$

$$= \frac{(2m)!u_e^{2m}}{4^m(m!)^2}\left[\ln\frac{1+\sin\theta}{\cos\theta} + \frac{\sin\theta}{\cos^2\theta}\sum_{k=0}^{m-1}\frac{4^k(k!)^2}{(2k+1)!\cos^{2k}\theta}\right]_0^{\theta_0}$$

$$= \frac{(2m)!u_e^{2m}}{4^m(m!)^2}\left[\ln\frac{1+\sin\theta_0}{\cos\theta_0} + \frac{\sin\theta_0}{\cos^2\theta_0}\sum_{k=0}^{m-1}\frac{4^k(k!)^2}{(2k+1)!\cos^{2k}\theta_0}\right]$$

$$= \frac{(2m)!u_e^{2m}}{4^m(m!)^2}\left[-\ln\frac{u_e}{u_0+\sqrt{u_0^2-u_e^2}} + \sqrt{u_0^2-u_e^2}\frac{u_0}{u_e^2}\sum_{k=0}^{m-1}\frac{4^k(k!)^2u_0^{2k}}{(2k+1)!u_e^{2k}}\right]$$

or

$$I_m = \frac{(2m)!u_e^{2m}}{4^m(m!)^2}\left[-\ln\frac{u_e}{u_0+\sqrt{u_0^2-u_e^2}} + \sqrt{u_0^2-u_e^2}\frac{u_0}{u_e^2}\sum_{k=0}^{m}\frac{4^k(k!)^2u_0^{2k}}{(2k+1)!u_e^{2k}}(1-\delta_{k,m})\right], \quad (m=0,1,2,\dots) \tag{17}$$

where

$$\delta_{k,m} = \frac{1-(-1)^{(|m-k|+1)\cdot !}}{2} = \begin{cases}1, & k=m,\\ 0, & k\neq m.\end{cases}$$

Second, we find V'. Owing to

$$d\left(2\sqrt{\cosh y - \cosh y_e}\frac{\sqrt{\cosh y+1}}{\sqrt{\cosh y-1}}\right)$$
$$= \frac{\sqrt{\cosh y-\cosh y_e}}{\sqrt{\cosh^2 y-1}}\mathrm{d}\cosh y + \frac{(\cosh y_e-1)\,\mathrm{d}\cosh y}{\sqrt{\cosh y-\cosh y_e}\sqrt{\cosh^2 y-1}}$$
$$+\frac{2(\cosh y_e-1)}{(\cosh y-1)\sqrt{\cosh y-\cosh y_e}}\frac{\mathrm{d}\cosh y}{\sqrt{\cosh^2 y-1}}. \tag{18}$$

Replacing $\frac{\sqrt{\cosh y-\cosh y_e}}{\sqrt{\cosh^2 y-1}}\mathrm{d}\cosh y$ in equation (10) with equation (18), we obtain

$$V' = 4\sqrt{2}\left(\sqrt{\cosh y_0+1}-\sqrt{2}\right) - 4\sqrt{2}\sqrt{\cosh y_0-\cosh y_e}\frac{\sqrt{\cosh y_0+1}}{\sqrt{\cosh y_0-1}}$$
$$+\int_{\cosh y_e}^{\cosh y_0}\frac{\sqrt{2}(\cosh y_e-1)\,\mathrm{d}\cosh y}{\sqrt{\cosh^2 y-1}\sqrt{\cosh y-\cosh y_e}}$$
$$+\int_{\cosh y_e}^{\cosh y_0}\frac{4\sqrt{2}(\cosh y_e-1)}{(\cosh y-1)\sqrt{\cosh y-\cosh y_e}}\frac{\mathrm{d}\cosh y}{\sqrt{\cosh^2 y-1}}. \tag{19}$$

Substituting equations (11) and (12) into (19), we obtain

$$V' = \frac{16u_0^2}{1-u_0^2} - \frac{8\left(1+u_0^2\right)\sqrt{u_0^2-u_e^2}\sqrt{1-u_0^2u_e^2}}{u_0\left(1-u_e^2\right)\left(1-u_0^2\right)} + \frac{8u_e^2}{1-u_e^2}\int_{u_e}^{u_0}\frac{\mathrm{d}u}{u^2\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}} + \frac{8u_e^2}{1-u_e^2}\int_{u_e}^{u_0}\frac{u^2\mathrm{d}u}{\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}}, \quad (20)$$

we have yet

$$\mathrm{d}\left(\frac{1}{u}\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}\right) = \frac{u_e^2\mathrm{d}u}{u^2\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}} - \frac{u_e^2u^2\mathrm{d}u}{\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}}. \quad (21)$$

Replacing $\frac{\mathrm{d}u}{u^2\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}}$ in equation (20) with equation (21), we obtain

$$V' = \frac{16u_0^2}{1-u_0^2} - 16u_0\frac{\sqrt{u_0^2-u_e^2}\sqrt{1-u_0^2u_e^2}}{\left(1-u_0^2\right)\left(1-u_e^2\right)} + \frac{16u_e^2}{1-u_e^2}\int_{u_e}^{u_0}\frac{u^2\mathrm{d}u}{\sqrt{u^2-u_e^2}\sqrt{1-u_e^2u^2}}. \quad (22)$$

By equation (22) we know that when $u_e = u_0$,

$$V' = \frac{16u_0^2}{1-u_0^2};\ \text{ when } u_e = 0,\quad V' = 0.$$

Expanding equation (22) in the series, and retaining the first n terms, we obtain approximate expression

$$V_n' = \frac{16u_0^2}{1-u_0^2} - 16u_0\frac{\sqrt{u_0^2-u_e^2}\sqrt{1-u_0^2u_e^2}}{\left(1-u_0^2\right)\left(1-u_e^2\right)} + \frac{16u_e^2}{1-u_e^2}\sum_{m=0}^{n}\frac{(2m)!u_e^{2m}}{4^m\,(m!)^2}I_{m+1}. \quad (23)$$

Equation (23) is just the expression used to calculate the interaction energy between two similar plane parallel double layers.

In order to compare with the results of the literature [3], we give another expression used to calculate the interaction energy in the literature [3] as follows:

$$V_o' = 8u_0^4\left[2\left(1+u_0^2+u_0^4\right)\left(1-\tanh\frac{\xi_d}{2}\right)\frac{1}{u_0^2} - \left(1+u_0^2\right)\frac{\sinh\frac{\xi_d}{2}}{\cosh^3\frac{\xi_d}{2}} - \frac{\frac{\xi_d}{2}}{\cosh^4\frac{\xi_d}{2}} - \frac{u_0^2}{2}\left(\frac{\sinh\frac{\xi_d}{2}}{\cosh^5\frac{\xi_d}{2}} + \frac{5\frac{\xi_d}{2}}{\cosh^6\frac{\xi_d}{2}} - \frac{\xi_d^2\sinh\frac{\xi_d}{2}}{\cosh^7\frac{\xi_d}{2}}\right)\right]. \quad (24)$$

By equation (24) we know that when $\xi_d = 0\,(u_e = u_0)$,

$$V_o' = 16u_0^2\left(1+u_0^2+u_0^4\right)$$

Table 1
The interaction energies between two parallel flat plates at various potential and distances.

$-C$	$\xi_{d,n}$	V'_o	V'_n	V'_{lite}
$y_0 = 1.00000 \times 10^{-1}$				
2.00004	6.90532 (0)	2.00202×10^{-5}	2.00202×10^{-5} (0)	4.82798×10^{-5}
2.00043	4.51050 (0)	2.17369×10^{-4}	2.17369×10^{-4} (0)	2.45929×10^{-4}
2.00181	2.99766 (0)	9.50168×10^{-4}	9.50168×10^{-4} (0)	9.64284×10^{-4}
2.00359	2.20025 (0)	1.99364×10^{-3}	1.99364×10^{-3} (0)	2.00391×10^{-3}
2.00635	1.39939 (0)	3.95700×10^{-3}	3.95700×10^{-3} (0)	3.95620×10^{-3}
2.00998	1.06387×10^{-1} (0)	9.47021×10^{-3}	9.47021×10^{-3} (0)	9.49705×10^{-3}
$y_0=2.00000\times10^{-1}$				
2.00002	8.98527 (0)	1.00013×10^{-5}	1.00013×10^{-5} (0)	1.10865×10^{-5}
2.00721	3.00060 (0)	3.78446×10^{-3}	3.78446×10^{-3} (0)	3.79026×10^{-3}
2.01435	2.19976 (0)	7.96698×10^{-3}	7.96698×10^{-3} (0)	7.95162×10^{-3}
2.02539	1.40024 (0)	1.58022×10^{-2}	1.58022×10^{-2} (0)	1.58044×10^{-2}
2.04010	5.75209×10^{-2} (1)	3.88794×10^{-2}	3.88794×10^{-2} (0)	3.88820×10^{-2}
$y_0=6.00000\times10^{-1}$				
2.00006	1.00708×10 (0)	3.00017×10^{-5}	3.00017×10^{-5} (0)	3.26633×10^{-5}
2.01538	4.50002 (1)	7.78232×10^{-3}	7.78232×10^{-3} (0)	7.76851×10^{-3}
2.06391	3.00000 (1)	3.35521×10^{-2}	3.35521×10^{-2} (0)	3.35543×10^{-2}
2.12764	2.19998 (1)	7.06403×10^{-2}	7.06403×10^{-2} (1)	7.06396×10^{-2}
2.22878	1.39999 (1)	1.40813×10^{-1}	1.40813×10^{-1} (1)	1.40823×10^{-1}
2.37092	1.01487×10^{-2} (1)	3.60822×10^{-1}	3.60826×10^{-1} (0)	3.60912×10^{-1}
$y_0=1.00000$				
2.00017	1.00248×10 (0)	8.50071×10^{-5}	8.50071×10^{-5} (0)	9.02414×10^{-5}
2.04146	4.50002 (1)	2.10098×10^{-2}	2.10097×10^{-2} (0)	2.10140×10^{-2}
2.17215	3.00000 (1)	9.04003×10^{-2}	9.03997×10^{-2} (1)	9.04027×10^{-2}
2.34649	2.19999 (1)	1.90636×10^{-1}	1.90636×10^{-1} (1)	1.90636×10^{-1}
2.63598	1.40001 (1)	3.83309×10^{-1}	3.83311×10^{-1} (1)	3.83308×10^{-1}
3.08615	5.71313×10^{-3} (2)	1.01769	1.01791 (1)	1.01835
$y_0=2.20000$				
2.00073	9.99668 (0)	3.65078×10^{-4}	3.65078×10^{-4} (0)	3.65973×10^{-4}
2.17009	4.49998 (1)	8.67770×10^{-2}	8.67707×10^{-2} (1)	8.67720×10^{-2}
2.70488	3.00000 (2)	3.70396×10^{-1}	3.70225×10^{-1} (2)	3.70225×10^{-1}
3.76145	2.00000 (2)	9.48054×10^{-1}	9.48145×10^{-1} (3)	9.48149×10^{-1}
5.58307	1.20000 (3)	1.97697	1.97733 (2)	1.97733
9.13569	5.05007×10^{-3}(4)	5.24696	5.33013 (2)	5.33043
$y_0=3.00000$				
2.00001	1.47640×10 (0)	5.00002×10^{-6}	5.00002×10^{-6} (0)	1.07288×10^{-6}
2.43613	4.00000 (2)	2.25437×10^{-1}	2.25295×10^{-1} (2)	2.25297×10^{-1}
3.63557	2.60000 (3)	8.62318×10^{-1}	8.60808×10^{-1} (3)	8.60810×10^{-1}
6.32785	1.60000 (3)	2.23236	2.23667 (3)	2.23668
1.10205×10	8.99997×10^{-1}(5)	4.46128	4.46448(4)	4.46447
2.01347×10	4.98715×10^{-3}(6)	1.00720×10	1.07741×10 (3)	1.07740×10

Table 1 *(Continued)*

$-C$	$\xi_{d,n}$	V'_o	V'_n	V'_{lite}
y_0=4.00000				
2.00001	1.51271×10 (0)	5.00002×10^{-6}	5.00002×10^{-6} (0)	5.24521×10^{-6}
2.61769	4.00000 (2)	3.21018×10^{-1}	3.20448×10^{-1} (2)	3.20445×10^{-1}
5.50009	2.20000 (3)	1.80403	1.80105 (3)	1.80105
1.04198×10	1.40000(6)	3.99388	4.01609(5)	4.01607
2.37467×10	6.99999×10^{-1}(7)	8.72386	8.83795(7)	8.83793
5.46118×10	5.00622×10^{-3} (12)	1.77119×10	2.19659×10 (6)	2.19661×10
y_0=5.00000				
2.00001	1.53427×10 (0)	5.00002×10^{-6}	5.00002×10^{-6} (0)	6.67572×10^{-6}
3.21876	3.50000 (3)	6.41013×10^{-1}	6.37116×10^{-1} (3)	6.37108×10^{-1}
7.46947	2.00000(4)	2.71641	2.72163(5)	2.72163
2.21493×10	9.99999×10^{-1}(8)	8.08256	8.12592(8)	8.12590
5.43315×10	5.00000×10^{-1} (11)	1.51015×10	1.63275×10 (11)	1.63275×10
1.48385×10^2	5.03581×10^{-3}(17)	2.56441×10	4.06897×10 (11)	4.06924×10

this is just the first 3 terms of the expansion of $16u_0^2/1-u_0^2$; when $\xi_d \to +\infty$ $(u_e = 0)$, $V'_o = 0$.

ξ_d in equation (24) is calculated by Equation (15).

3. The result and discussion

We list the numerical results of equations (15), (23) and (24) in table 1. The numbers on the rightmost column in table 1 are the result of the literature [4]. In table 1, (0) denote $n \geqslant 0$, (1) denote $n \geqslant 1$, (2) denote $n \geqslant 2$, and so on. By table 1 we know that with the first four terms of the series a very high precision with six significant figures can be reached when the dimensionless surface potential of the colloidal particle is less or equal to 3. When y_0 is less, the difference of V' and V'_{lite} comes from inaccurate V'_{lite}, because V' is very consistent with V'_o.

4. Conclusion

The interaction energy between two similar plane parallel double layers is expanded in a series of $\tanh(y_0/4)$ and a general expression is introduced. The series converges very fast at the moderate and low potentials. With the first four terms of the series a very high precision with six significant figures can be reached when the dimensionless surface potential of the colloidal particles is less or equal to 3.

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